EFFECT OF INITIAL FLUIDIZING-AGENT DISTRIBUTION ON THE LARGE-SCALE MOTION IN A HOMOGENEOUS FLUIDIZED BED

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On the basis of the idea of the phases of a fluidized bed as coexisting continuous media, the mean motion of the phases established with different initial distributions of the fluidizing medium is investigated.

The intensity of mixing and various bulk processes in a fluidized bed, and hence also the efficiency of operation of equipment based on such beds, is determined by the features of the small-scale random pulsations of the particles and fluid and also by their large-scale ordered flows. The form and level of development of the two types of motion and the local structure of the bed depend very strongly on the equipment used to introduce the fluidizing agent in the bed and, in particular, on its initial velocity profile. Although much empirical information has been accumulated on the relation between the characteristics of the distributing agent and the observed hydrodynamic properties of the fluidized bed, the theoretical generalization necessary for rational control of the bed structure and the motion of its phases does not actually exist.

A priori, two basic mechanisms by which the initial fluidizing-agent distribution will affect the resulting large-scale motion may be isolated. In significantly inhomogeneous beds, especially when the medium is introduced through nozzles or through the holes of a perforation, the bed structure and phase motion is mainly determined by the motion and coalescence of the interacting bubbles initially formed in the near-lattice region — for example, at the individual elementary jets issuing from nozzles or holes. In the bed, there appear stable regions of preliminary bubble ascent and regions depleted of bubbles, in which, respectively, the maximum and minimum fluidizing-agent velocities are reached and ascending or descending motion of the disperse phase is realized (see [1-4], for example). With increase in height above the initial cross section, the individual bubbles merge, their total number decreases, and the fluidizing-agent velocity distribution characteristic of the lower levels, with a few maxima, changes to a profile with a single maximum close to the axis of the apparatus [4]; in sufficiently narrow equipment, the subsequent development of a piston mode is possible [3]. Analysis of these profiles in various cross sections and the associated disperse-phase circulation in this case may be conducted on the basis of a theory of the evolution of the diluted phase of the fluidized bed, taking bubble-coalescence into account [5].

The other case, which is no less important in practice, is that typical of near-homogeneous beds, where the establishment of large-scale motion of the phases is due primarily to the retarding effect of the equipment walls on the flow of fluidizing medium, and occurs in principle in the same way as the development of a flux of single-phase viscous fluid in channels. In this case, regardless of the form of the initial fluidizing-medium velocity profile, a characteristic dome-shaped, near-parabolic profile is established in the limit, i.e., at sufficiently large distances from the inlet cross section [6-8]. The theory of the development of such a flow is outlined below.

Continuum Model of Bed Phases

In analyzing motion with a characteristic spatial scale considerably exceeding the size of the bed particles and the mean distance between them, it is natural to describe the continuous and disperse phases as two interpenetrating continuous media. The equations of mass and momentum conservation for these media in steady conditions may be written in the form [9]

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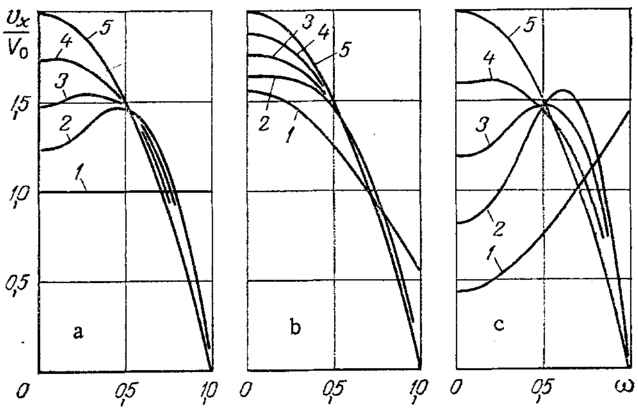

Fig. 1. Distribution of the dimensionless bulk velocity of the fluidizing medium in cross sections of the fluidized bed corresponding to the initial distribution in Eq. (11) with $A_1 = 0$ (a), 1 (b), −1 (c); $\exp\cdot(-\beta_1^2\xi/\mathrm{Re}) = 1$ (1), 0.6 (2), 0.4 (3), 0.2 (4), and 0 (5).

$$d_0\varepsilon(\mathbf{c}_0\nabla)\mathbf{c}_0 = \nabla\sigma_0 - \mathbf{f} + d_0\varepsilon\mathbf{g},\ \operatorname{div}(\varepsilon\mathbf{c}_0) = 0,$$
$$d_1\rho(\mathbf{c}_1\nabla)\mathbf{c}_1 = \nabla\sigma_1 + \mathbf{f} + d_1\rho\mathbf{g},\ \operatorname{div}(\rho\mathbf{c}_1) = 0,\ \rho = 1 - \varepsilon. \quad (1)$$

Note that investigators considering a fluidized bed on the basis of the continuum approximation have proceeded from particular variants of Eq. (1) which differ in the empirical representations chosen for the effective stress tensors σ_0 and σ_1 and the phase-interaction force f (see [10-12], for example).

It is possible to obtain a rigorous representation for these quantities only in the case of very small particles, when the Reynolds number characterizing their flow is of the order of unity or less [9]. Here, where the theory is to be used also for large-particle beds, these quantities will be specified semiempirically, starting from a series of simplifying assumptions.

First of all, the bed porosity ε is assumed to be uniform over the whole bed volume. If the initial nonuniformity of the fluidizing-agent distribution is not too large, this assumption is very close to the truth. However, as discussed in detail in [13], it necessitates the introduction of a new independent variable: the pressure p_1 of the medium modeling the disperse phase. Further, by analogy with the model of an ideal single-phase fluid, the pseudoviscous stress in both phases due to their pseudoturbulent pulsations is neglected in the first approximation. Then the media modeling the disperse and continuous phases must be regarded, respectively, as an ideal and a viscous Newtonian fluid, i.e., as satisfying the following relations

$$\nabla\sigma_0 = -\nabla p_0 + \mu\Delta\mathbf{c}_0,\ \nabla\sigma_1 = -\nabla p_1, \quad (2)$$

where μ is some effective viscosity coefficient. A model of this kind was considered earlier in [13]. In the limiting case opposite to Eq. (2), when the effective viscosity of the disperse phase is much larger than the viscosity of the continuous phase, it is possible to regard the particles as fixed in the first approximation and to investigate the filtration of the fluidizing medium in the porous body that they form [14].

The force f is represented in traditional form

$$\mathbf{f} = -\rho d\mathbf{g} + \rho k(\mathbf{c}_0 - \mathbf{c}_1),\ d = d_0\varepsilon + d_1\rho, \quad (3)$$

corresponding to a linear approximation of the phase-interaction force in the given range of relative phase velocity with empirical coefficient k.

In the inlet cross section of the bed, the distribution of the fluidizing-agent flow rate $v_x = \varepsilon c_{0x}$ is specified; the velocity c_{1x} in this cross section must obviously vanish. On the vertical walls of the apparatus, the velocity $\mathbf{c}_0$ and the horizontal components of the velocity $\mathbf{c}_1$ vanish. Thus, the effect of a thin near-wall layer and the resulting "slippage" of the fluidizing agent is neglected. Within the framework of the proposed model, when the disperse-phase viscosity is small and disregarded, this is completely justified, as shown in [14] in the case where the particle size is much less than the dimensions of the apparatus. Considering a cylindrical apparatus, so as to be specific, and assuming the flow to be axisymmetric, the boundary conditions are written as follows

$$\begin{gathered} \varepsilon c_{0x} = V_0[1 + F(r/R)],\ c_{1x} = 0,\ x = 0; \\ c_{0x} = c_{0r} = c_{1r} = 0,\ r = R;\ c_{0r} = c_{1r} = 0,\ r = 0, \end{gathered} \tag{4}$$

where the function F(r/R) is defined so that its integral over the cylinder cross section identically vanishes.

No boundary conditions are imposed on the initially unknown upper boundary of the bed, i.e., motion in an infinitely tall bed is actually considered. According to the data of [4, 7, 8], the effect of a free upper surface on the motion inside the layer is not very large. At the same time, taking it into account would considerably complicate the solution of the problem and is hardly expedient at present.

Motion of the Continuous Phase

Under the given simplifying assumptions, the boundary problem obtained for Eq. (1) with Eqs. (2) and (3) and the boundary conditions in Eq. (4) reduces to the simpler problem of the development of a flow of single-phase viscous liquid in a tube. To solve this problem, such methods are available as the Boussinesq, which leads to good results far from the inlet cross section but poor results closer, and the Schiller, which, conversely, leads to good results at the inlet but poor results further away [15]. Here, use is made of the more efficient method of S. M. Targ, which not only combines the advantages of the Boussinesq and Schiller methods, giving good agreement with experiment, but also considerably simplifies the calculations [15].

According to this method, it is assumed that $c_{0r} \ll c_{0x}$, $\partial^2 c_{0x}/\partial x^2 \ll \partial^2 c_{0x}/\partial r^2$, and the inertial terms on the left-hand side of the momentum-conservation equations in Eq. (1) are transformed in the manner of the well-known Ozeen approximation, replacing εc_{0x} by V_0 and c_{0r}, c_{1x}, and c_{1r} by zero. Then, adding the momentum-conservation equations of the two phases, it is found from the r components of the resulting equation that $p_0 + p_1$ depends only on x, while the x component of this equation is written in the form

$$d_0 V_0 \frac{\partial c_{0x}}{\partial x} = -\frac{d}{dx}(p_0 + p_1) + \mu\left(\frac{\partial^2 c_{0x}}{\partial r^2} + \frac{1}{r}\frac{\partial c_{0x}}{\partial r}\right) - dg. \tag{5}$$

In addition to Eq. (5), the mass-conservation equation of the continuous phase from Eq. (1) and the boundary conditions on c_0 from Eq. (4) are considered.

The dimensionless variables and parameters

$$\begin{gathered} \xi = \frac{x}{R},\ \omega = \frac{r}{R},\ V_x = \frac{\varepsilon c_{0x} - (1 + F) V_0}{V_0},\ V_r = \frac{\varepsilon c_{0r}}{V_0}, \\ P = \varepsilon \frac{p_0 + p_1 + dgx}{d_0 V_0^2} - P',\ \mathrm{Re} = \frac{V_0 R}{\nu},\ \nu = \frac{\mu}{d_0} \end{gathered} \tag{6}$$

are now introduced, and the constant P' is chosen so that P vanishes at x = 0. In the variables of Eq. (6), the autonomous problem for V_x, V_r, and P takes the form

$$\begin{gathered} \frac{\partial^2 V_x}{\partial \omega^2} + \frac{1}{\omega}\frac{\partial V_x}{\partial \omega} = \mathrm{Re}\left(\frac{\partial V_x}{\partial \xi} + \frac{dP}{d\xi}\right) - \frac{d^2 F}{d\omega^2} - \frac{1}{\omega}\frac{dF}{d\omega}; \\ \frac{\partial V_x}{\partial \xi} + \frac{1}{\omega}\frac{\partial(\omega V_r)}{\partial \omega} = 0;\ V_x = P = 0,\ \xi = 0; \\ V_x = -1 - F(1),\ V_r = 0,\ \omega = 1;\ V_r = 0,\ \omega = 0 \end{gathered} \tag{7}$$

and coincides with one of the problems considered in [15].

The function F(ω) appearing in Eqs. (4) and (7) is written in the form

$$F(\omega) = \sum_{n=1}^{\infty} A_n[J_0(\alpha_n \omega) - J_2(\alpha_n)], \tag{8}$$

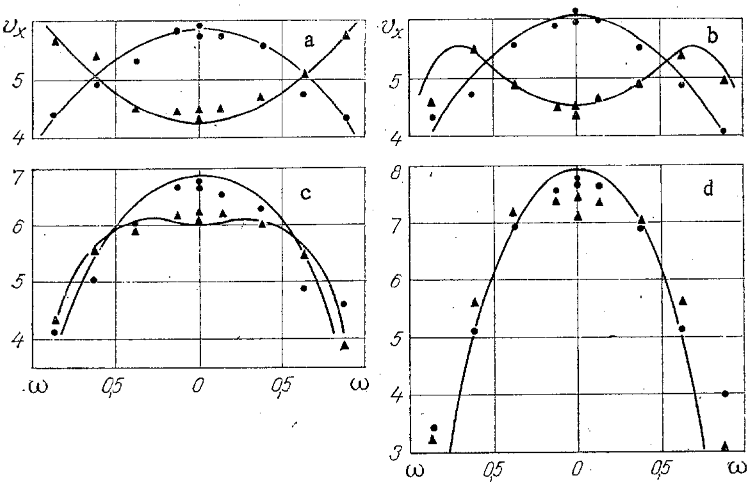

Fig. 2. Bulk velocity profile of gas in fluidized bed in cross sections at a distance of 36 (a), 56 (b), 96 (c), and 136 mm (d); the curves correspond to theory and the points to experimental data from [8], convex and concave initial distributions; points to the left and right of the axis $\omega = 0$ correspond to different experiments performed in identical conditions; d) theoretical limiting velocity profile. v_x, m/sec.

where α_n are the successive roots of the equation $J_0(x) = 0$, and $J_m(x)$ are Bessel functions. For practical purposes, it is evidently sufficient to approximate the real initial velocity profile of the fluidizing medium by only the first few terms of the series in Eq. (8).

Omitting the details of the fairly lengthy computations, which are available in [15], the final expressions for P and v_x/V_0 are

$$P = \frac{8}{\mathrm{Re}}\frac{x}{R} + \frac{1}{3} + \sum_{n=1}^{\infty}\left(1 - \frac{8}{\alpha_n^2}\right)A_n J_2(\alpha_n) - 4\sum_{k=1}^{\infty}\left\{\frac{1}{\beta_k^2} + \sum_{n=1}^{\infty}\frac{A_n J_2(\alpha_n)}{\alpha_n^2 - \beta_k^2}\right\}\exp\left(-\frac{\beta_k^2}{\mathrm{Re}}\frac{x}{R}\right),$$

$$\frac{v_x}{V_0} = 2\left(1 - \frac{r^2}{R^2}\right) - 4\sum_{k=1}^{\infty}\left\{\frac{1}{\beta_k^2} + \sum_{n=1}^{\infty}\frac{A_n J_2(\alpha_n)}{\alpha_n^2 - \beta_k^2}\right\}\left\{1 - \frac{J_0(\beta_k r/R)}{J_0(\beta_k)}\right\}\exp\left(-\frac{\beta_k^2}{\mathrm{Re}}\frac{x}{R}\right), \tag{9}$$

where β_k are the successive roots of the equation $J_2(x) = 0$, and the factors A_n are presumed known.

Integration of the second relation in Eq. (7), taking the expression for v_x/V_0 in Eq. (9) into account, gives

$$\frac{v_r}{V_0} = -\frac{4}{\mathrm{Re}}\sum_{k=1}^{\infty}\left\{1 + \beta_k^2\sum_{n=1}^{\infty}\frac{A_n J_2(\alpha_n)}{\alpha_n^2 - \beta_k^2}\right\}\left\{\frac{1}{2}\frac{r}{R} - \frac{1}{\beta_k}\frac{J_1(\beta_k r/R)}{J_0(\beta_k)}\right\}\exp\left(-\frac{\beta_k^2}{\mathrm{Re}}\frac{x}{R}\right). \tag{10}$$

This formula describes the intensity of the radial flow of fluidizing agent to the axis of the cylindrical equipment.

Thus, the limiting fluidizing-agent velocity profile does not depend on its initial distribution and corresponds to Poiseuille flow, and its development with increasing distance from the inlet cross section conforms to the same law as in the case of a single-phase fluid. Note that the expressions for v_x and v_r are universal in the sense that they do not depend on the coefficient of hydrodynamic phase interaction, k, and moreover they do not change if the approximation in Eq. (3) is replaced by any other formula for f.

The establishment of the limiting axial-velocity profile for the fluidizing agent is shown in Fig. 1 in the case of an initial profile of the form

$$\left.\frac{v_x}{V_0}\right|_{x=0} = 1 + A_1\left[J_0\left(\alpha_1\frac{r}{R}\right) - J_2(\alpha_1)\right] \tag{11}$$

for different values of A_1. Since the terms of the series for v_x/V_0 in Eq. (9) rapidly diminish, only the term with $k = 1$ is calculated. In Fig. 2, as an example, the theory is compared with two of the experiments in [8] for round particles of alumosilicate catalyst of diameter $(3.5\text{–}4)\cdot 10^{-3}$ m fluidized by air in an apparatus of diameter $D = 0.172$ m, with fluidization number $N = 4$ and a motionless-filling height $H = 0.75D = 0.129$ m. The initial velocity profile determined experimentally is approximated by Eq. (11), and Re is found by trial and error. As is evident from Fig. 2, agreement between theory and experiment is satisfactory. Comparison with other experiments in [8] on the whole confirms this conclusion.

However, it should be noted that at fluidization numbers $N = 1\text{–}2$ the experimental velocity profiles for air are more sloping than would follow from the theory. This is evidently because in these conditions there are direct mechanical interactions between the particles, resulting in an effective viscosity of the disperse phase, which was neglected in the theory. In fact, it follows from [14] that increase in this viscosity leads to compression of the profile in the central part of the apparatus in comparison with the parabolic form. In the general case, the true profile must lie between the Poiseuille parabolic profile and an almost plane profile (the latter is realized for filtration in a fixed charge of particles and is discussed in detail in [14]).

In addition, the observed compression of the velocity profile may be associated with the effect of the free upper surface of the bed on the flow and also with the onset of significant bubble formation. In fact, a chain of bubbles rising one after the other into the upper part of the bed may be roughly interpreted as the flare of a two-phase jet in which the distribution is approximately described by a Schlichting profile much steeper than the parabolic Poiseuille profile. This agrees with the ideas of the so-called "flare" approach adopted in [4, 16].

Motion of the Disperse Phase

Taking account of Eqs. (2) and (3) and the assumptions made above, the components of the momentum-conservation equation for the disperse phase from Eq. (1) are written in the form

$$-\partial p_1/\partial x + \rho k(c_{0x} - c_{1x}) - (d_1 - d_0)\rho g = 0, \quad -\partial p_1/\partial r + \rho k(c_{0r} - c_{1r}) = 0. \tag{12}$$

If the operators $\partial/\partial x$ and $r^{-1}(\partial/\partial r)r$, respectively, act on these equations, and the results are added, taking the mass-conservation equation in Eq. (1) into account, the Laplace equation for p_1 is obtained. The boundary conditions which must be imposed on p_1 are obtained from the first and second relations in Eq. (12), taking account of Eq. (4) written at $x = 0$ and $r = R$, respectively, and the condition of axial symmetry. Thus, in the variables of Eq. (6), the following problem for the dimensionless pressure of the disperse phase is obtained

$$\frac{\partial^2 P_1}{\partial \xi^2} + \frac{1}{\omega}\frac{\partial}{\partial \omega}\left(\omega \frac{\partial P_1}{\partial \omega}\right) = 0, \quad P_1 = \frac{\varepsilon p_1}{d_0 V_0^2};$$
$$\frac{\partial P_1}{\partial \xi} = \rho \frac{R}{d_0 V_0}\left\{k[1 + F(\omega)] - \varepsilon(d_1 - d)\frac{g}{V_0}\right\} = \rho \Pi(\omega), \quad \xi = 0; \tag{13}$$
$$\frac{\partial P_1}{\partial \omega} = 0, \quad \omega = 0; \quad \frac{\partial P_1}{\partial \omega} = 0, \quad \omega = 1.$$

Using the method of separation of variables, a more general solution satisfying the conditions at $\omega = 0$ and $\omega = 1$ is written, in the form

$$P_1 = \rho\left\{B_0 \xi + \sum_{n=1}^{\infty} B_n J_0(\gamma_n \omega)\exp(-\gamma_n \xi)\right\} \tag{14}$$

(insignificant constants in determining P_1 are omitted), where γ_n are the successive roots of the equation $J_0'(x) \equiv J_1(x) = 0$. The series corresponding to Eq. (14) for the derivative $\partial P_1/\partial \xi$ at $\xi = 0$ consists of the Din–Bessel expansion of the function $\Pi(\omega)$; in other words

$$B_0 = -2\int_0^1 \omega \Pi(\omega)\, d\omega, \quad B_n = -\frac{2}{\gamma_n}\int_0^1 \omega \Pi(\omega) J_0(\gamma_n \omega)\, d\omega, \quad n \geqslant 1. \tag{15}$$

When $\xi \to \infty$, P_1 must remain finite, and therefore $B_0 = 0$ is necessary. Taking account of the condition imposed on the function $F(\omega)$ when it was introduced in the boundary conditions in Eq. (4), it is evident that this requires that the following relation hold

$$k(V_0/\varepsilon) = (d_1 - d)\,g, \tag{16}$$

this being, essentially, the usual equation for determining the porosity constant of a bed which has previously been unknown. In view of Eq. (16), the definition of $\Pi(\omega)$ in Eq. (13) yields

$$\Pi(\omega) = (\rho k R/d_0 V_0)\,F(\omega) \tag{17}$$

and further

$$B_n = \frac{\rho k R}{d_0 V_0} b_n,\; b_n = -\frac{2}{\gamma_n}\int_0^1 \omega F(\omega)\,J_0(\gamma_n\omega)\,d\omega, \tag{18}$$

which finally determines the value of P_1 in Eq. (14).

Calculating the derivatives $\partial p_1/\partial x$ and $\partial p_1/\partial r$ on the basis of the relations given above, and using Eqs. (12) and (18), expressions are obtained for the components of the relative phase velocity

$$u_x = c_{0x} - c_{1x} = U - \frac{V_0}{\varepsilon}\sum_{n=1}^{\infty}\gamma_n b_n J_0(\gamma_n\omega)\exp(-\gamma_n\xi),$$
$$u_r = c_{0r} - c_{1r} = \frac{V_0}{\varepsilon}\sum_{n=1}^{\infty}\gamma_n b_n J_1(\gamma_n\omega)\exp(-\gamma_n\xi),\; U = \frac{(d_1 - d)\,g}{k}. \tag{19}$$

Equations (9), (10), and (19) also allow the components of the mean disperse-phase velocity $\mathbf{c}_1$ to be determined, and Eqs. (9) and (14) allow the distribution of the static fluidizing-agent pressure p_0 in the bed to be found.

For uniform initial distribution of the fluidizing medium, the relative phase velocity is uniform over the whole bed volume ($u_x = U$, $u_r = 0$), and $p_1 = 0$, so that the pressure p_0 is easily found from Eqs. (6) and (9) and it is seen to be independent of r. In this case, there is a single circulation contour of the disperse phase, with upward motion at the axis of the apparatus and downward motion at the wall. With a nonuniform distribution, the fields of u and p_0 are nonuniform. If the excess of fluidizing medium is supplied close to the wall, there arises a second circulation contour of the disperse phase, occupying the lower part of the bed, in which the particles move downwards in regions close to the axis and directly at the wall and upwards between them. The quantitative characteristics of macroscopic motion of the two phases are not difficult to obtain, by investigating in detail the above relations.

In conclusion, the basic limitations of the theory proposed, removal of which would be desirable in its further development, will be enumerated. First, the effect of the "dilute" bubble phase on the distribution of flows in the bed is assumed to be weak. For this tc be the case, the bubble dimensions, and also their bulk concentration, must be small. It is clear that this assumption that the bubbles play an inconsiderable role is particularly easily violated in gas-fluidized beds at some distance from the gas-distributing lattice. To obviate this assumption, it would be necessary to introduce into the analysis some of the considerations discussed in [5].

Secondly, the effective viscosity of the disperse phase has been neglected, which restricts the application of the theory to situations where direct friction of the particles and their pseudoturbulent motion is relatively weak. Thirdly, the motion has in fact been investigated in an infinitely tall bed, and no account at all has been taken of the stabilizing influence of the free upper surface of the bed. This type of fluidized bed is formed, for example, in the removal of crushed rock in drilling oil and gas wells [17] and in a number of other circumstances. However, in most cases that are of practical interest, the height of the fluidized bed is commensurate with its transverse dimensions, and the assumption made cannot, generally speaking, be regarded as correct. In such cases, the limiting profiles of the mean phase velocity may simply not be established, and a better description of the fluidizing-agent motion will possibly be obtained from the solution of the filtration problem within the framework of the assumption that the disperse phase is on average almost motionless. To remove these limitations would require significant modification in the formulation of the problem. In the first case, it would be necessary to introduce an effective viscous stress tensor in the momentum-conservation equation of the disperse phase and to assume that the viscous stress in both phases may be anisotropic (the phases themselves are regarded as non-Newtonian viscous fluids). In the second case it would be necessary to consider the problem with an unknown upper bed boundary or, if the boundary is assumed to be known from any additional considerations, to impose on this boundary the condition that p_1 and the normal velocity component $\mathbf{c}_1$ vanish.

NOTATION

A_n, B_n, b_n, coefficients of the series in Eqs. (8) and (18); $\mathbf{c}$, mean velocity; d, density; F, function introduced in Eq. (4); $\mathbf{f}$, phase-interaction force; $\mathbf{g}$, acceleration due to gravity; k, phase-interaction coefficient; p, P, dimensional and dimensionless pressure; R, radius of apparatus; r, radial coordinate; $\mathbf{u}$, U, relative gas velocity and limiting value of its modulus; $\mathbf{v}$, $\mathbf{V}$, dimensional and dimensionless bulk velocity of fluidizing agent; V_0, mean value of v_{0x} in the initial cross section; x, axial coordinate; α_n, β_n, γ_n, roots of Bessel functions; ε, porosity; μ, ν, effective dynamic and kinematic viscosity of fluidizing agents; $\xi = x/R$; Π, function introduced in Eq. (13); $\rho = 1 - \varepsilon$; σ, effective stress tensor; $\omega = r/R$; the subscripts 0 and 1 correspond to continuous and disperse phases of the bed; $Re = V_0R/\nu$.

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